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THEORY OF HEAT TRANSFER AND HYDRAULIC RESISTANCE  
OF OIL RADIATORS

By N. B. Mariamov

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THEORY OF HEAT TRANSFER AND HYDRAULIC RESISTANCE  
OF OIL RADIATORS\*

By N. B. Mariamov

In the present report the coefficients of heat transfer and hydraulic resistance are theoretically obtained for the case of laminar flow of a heated viscous liquid in a narrow rectangular channel. The results obtained are applied to the computation of oil radiators, which to a first approximation may be considered as made up of a system of such channels. In conclusion, a comparison is given of the theoretical with the experimental results obtained from tests on airplane oil radiators.

NOTATION EMPLOYED

$\bar{v}$  mean velocity of flow of liquid

$v$  velocity of flow at any point

$t_0$  temperature of the wall

$\bar{t}$  mean temperature of the liquid at any cross section

$t$  temperature of the liquid at any point

$t_i$  temperature of the liquid at inlet to channel or radiator

$\bar{\theta} = \bar{t} - t_0$  mean temperature difference at any cross section

$\theta = t - t_0$  temperature difference at any point

$\theta_0 = t_i - t_0$  temperature difference at inlet to channel or radiator

$a$  heat conduction coefficient

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\*Report No. 444, of the Central Aero-Hydrodynamical Institute, Moscow, 1939.

- $\lambda$       coefficient of heat conductivity of liquid
- $\mu$       coefficient of viscosity of liquid
- $h$       height (width) of channel (passage) through which liquid flows
- $d_h$       hydraulic diameter of passage
- $L$       length of channel through which liquid flows
- $Nu$       Nusselt number
- $Pe_h$       Péclet number referred to hydraulic diameter
- $Re_h$       Reynolds number referred to hydraulic diameter
- $\lambda_{fr}$       coefficient of hydraulic resistance

## INTRODUCTION

The problem of oil cooling of airplane engines is a very important one at the present time. Only a comparatively short time ago this was not the case because of the relatively small quantities of heat required to be dissipated by the radiator which consequently could be of small dimensions. Engine cooling by liquids with high boiling points considerably increases the heat transfer in the oil (1.5 to 2 times) a fact which leads to an increase in the required radiator dimensions. At the same time the aerodynamic fineness of airplanes has reached a stage where the air-oil radiator, which projects above the surface of the airplane, considerably impairs its aerodynamic efficiency. A study of the operation of oil radiators, with the object of obtaining computation formulas required for a rational design of the radiator under given conditions (speed of airplane, temperature of air and oil, power of engine, etc.), thus assumes considerable importance.

The main object of the present paper is that of determining the coefficients of heat transfer of oil radiators and their hydraulic resistance to the flow of the oil. Since the coefficient of heat transfer from the wall to the air is sufficiently well known (references 1,2) the problem of determining the heat transfer coefficients of oil radiators reduces to that of determining the heat transfer coefficients from the oil to the wall.

The large viscosity of aviation oils on the one hand, and the small width of the oil-flow passages and small velocity of flow on the other hand, permit us to consider the flow of the oil within the radiator as laminar. Under this assumption, we have attempted to determine theoretically the heat transfer coefficient from the oil to the wall.

The problem of heat transfer of a straight round tube, through which flows a heated viscous liquid, was first solved by Graetz and later by a number of other investigators. (See references.) Notwithstanding the rather rough assumptions which were generally made in the solution of this problem, the results for small values of the Reynolds number, as Kraussold has shown (fig. 1), are in sufficiently good agreement with experiment. The application, however, of these results to the computation of the heat-transfer coefficients of oil radiators may lead to large errors, for it is not known in what manner the data obtained for the round tube may be generalized to the narrow rectangular channels, such as provided by the passages for the liquid in airplane radiators with hexagonal tubes. For this reason, following the method of Nusselt, we have presented the solution of the problem for a narrow rectangular channel so that the solution may be applicable to a radiator (fig. 2) which, to a first approximation, may be considered as a system of such channels.

To simplify the solution we have made a number of assumptions not entirely corresponding to the actual process of heat transfer in the radiator. As will be shown below, however, we obtain in the given case, as also in the case of the round tube, a satisfactory agreement of the analytical solution with test results.

The obtained solution evidently may be applied to radiators with any viscous fluid as coolants, provided the flow is laminar.

## 2. HEAT TRANSFER FOR LAMINAR FLOW OF VISCOUS FLUID IN NARROW RECTANGULAR CHANNEL

Let us consider the flow of a heated viscous fluid in a straight channel having for its cross section a rectangle whose base is large relative to its height (fig. 3).

We shall choose the position of the coordinate axes as indicated on the figure. The motion of the liquid may be considered as two-dimensional since the side walls, in view of their distance from the  $x$ -axis, have only a slight effect on the velocity field.

The problem is that of determining the manner in which the coefficient of the heat transfer from the liquid to the walls of the channel varies with the velocity of flow and with the geometric parameters of the channel ( $h, L$ ). For the solution of our problem, we make the following assumptions:

- 1) The temperature of the channel walls is constant.
- 2) The channel wall surfaces are absolutely smooth.
- 3) The physical constants of the liquid at any cross section are constant.
- 4) The temperature of the liquid at the inlet section of the channel is the same at all points.
- 5) The laminar flow of the liquid is fully developed over the entire part of the channel under consideration.

Since laminar flow of the liquid is assumed, the phenomenon of heat transfer from the liquid to the wall will be determined chiefly by the conduction of the heat in a direction normal to the motion of the fluid. With the coordinate axes chosen as in figure 3, we write down the fundamental differential equation of the steady temperature field

$$a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) = \frac{\partial t}{\partial x} v_x + \frac{\partial t}{\partial y} v_y + \frac{\partial t}{\partial z} v_z \quad (1)$$

where  $a$  heat conduction coefficient

$t$  temperature of the fluid at any point

$x, y, z$  coordinates of any point

$v_x, v_y, v_z$  components of the fluid velocity at any point

Denoting the constant temperature of the channel walls by  $t_0$  and the temperature difference at a given point

by  $\theta = t - t_0$  we obtain from (1) the differential equation of steady heat exchange

$$a \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = \frac{\partial \theta}{\partial x} v_x + \frac{\partial \theta}{\partial y} v_y + \frac{\partial \theta}{\partial z} v_z \quad (2)$$

Since the laminar flow of the fluid is in the direction of the  $x$ -axis, we have, obviously

$$v_y = v_z = 0$$

Further, assuming  $\frac{\partial \theta}{\partial x} \approx \text{const}$ , that is, that the temperature gradient of the temperature difference of the fluid

varies only slightly in the direction of the  $x$ -axis, we

may neglect the term  $\frac{\partial^2 \theta}{\partial x^2}$  by comparison with the term  $\frac{\partial^2 \theta}{\partial y^2}$ .

Finally, we have  $\frac{\partial^2 \theta}{\partial z^2} = 0$ . The differential equation of

heat exchange under our conditions thus becomes

$$a \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial \theta}{\partial x} v_x \quad (3)$$

From elementary hydraulic considerations, it can easily be shown that for laminar flow in a channel the cross section of which is a rectangle with base large relative to its height, the velocity distribution follows the parabolic law as in the case of the round tube and is expressed by the formula

$$v_x = \frac{3}{2} \bar{v} \left( 1 - 4 \frac{y^2}{h^2} \right) \quad (4)$$

where  $\bar{v}$  is the mean velocity of the fluid

and  $h$  the height of the channel.

Substituting in (3) the value obtained for  $v_x$  and setting  $h/2 = b$  we shall have

$$a \frac{\partial^2 \theta}{\partial y^2} = \frac{3}{2} \bar{v} \left( 1 - \frac{y^2}{b^2} \right) \frac{\partial \theta}{\partial x} \quad (5)$$

The obtained differential equation determines the temperature difference as a function of the coordinates; the other magnitudes - physical constants of the fluid, relative length of channel ( $L/h$ ), and mean velocity of flow - appearing as parameters.

Following the method of Fourier, we represent the general integral of equation (5) in the form of a product of two functions:

$$\theta = \varphi(x) \psi(y) \quad (6)$$

one of which is a function of  $x$  alone and the other of  $y$  alone. Substituting in equation (5), we obtain:

$$a \frac{d^2 \psi}{dy^2} \varphi = \frac{3}{2} \bar{v} \left( 1 - \frac{y^2}{b^2} \right) \frac{d\varphi}{dx} \psi$$

or

$$\frac{\frac{d^2 \psi}{dy^2}}{\left( 1 - \frac{y^2}{b^2} \right) \psi} = \frac{\frac{3}{2} \bar{v} \frac{d\varphi}{dx}}{a\varphi} \quad (7)$$

In the above expression, the left side is a function of  $y$  alone and the right side of  $x$  alone. Since the equality is true for all values of  $x$  and  $y$  it follows that the right and left sides of the equation must be equal to the same constant magnitude. Denoting the latter

by  $-\frac{c^2}{b^2}$  the differential equation (5) breaks up into a pair of ordinary differential equations

$$\frac{d\varphi}{dx} + \frac{2}{3} \frac{ac^2}{\bar{v}b^2} \varphi = 0 \quad (8)$$

$$\frac{d^2 \psi}{dy^2} + \frac{c^2}{b^2} \left( 1 - \frac{y^2}{b^2} \right) \psi = 0 \quad (9)$$

The integral of equation (8) is

$$\varphi = De^{-\frac{2}{3} \frac{a}{\bar{v}} \frac{c^2}{b^2} x} \quad (11)$$

where  $D$  and  $c$  are constants determined by the following boundary conditions:

$\theta(0, y) = \theta_0$  condition at inlet section of channel

$\theta(x, b) = 0$  condition at surface of channel

By a change in the variable  $y/b = \xi$  equation (9) is transformed into

$$\frac{d^2\psi}{d\xi^2} + c^2(1 - \xi^2)\psi = 0 \quad (12)$$

The solution of the above equation is a certain function  $\psi(c^2; \xi)$ . By applying the method of successive approximations to equation (12), we can find this function. Assuming it as known, we obtain the general integral of equation (5) in the form

$$\theta = De^{-\frac{c^2}{b} \frac{2}{3} \frac{a}{v} x} \psi(c^2, \xi) \quad (13)$$

The constants of the integration  $c^2$  and  $D$  are determined from a consideration of the boundary conditions. The constant  $c^2$  is easily found from the second condition which, evidently, can be satisfied only if

$$\psi(c^2; 1) = 0 \quad (14)$$

In this way we arrived at the equation by which the constant  $c^2$  is determined.

The function  $\psi(c^2; \xi)$ , appearing as the solution of equation (12), cannot be represented in finite form and is expressed in the form of an infinite series. Equation (14) thus has an infinite multiplicity of roots. By applying the method of successive approximations, we computed the first three roots

$$\psi(c^2, 1)_1 = c_1^2 = 2.827$$

$$\psi(c^2, 1)_2 = c_2^2 = 32.0$$

$$\psi(c^2, 1)_3 = c_3^2 = 92.1$$

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Corresponding to these, expression (13) for the temperature difference may be represented in the form of an infinite sum of particular solutions

$$\theta = \sum_{i=1}^{i=\infty} D_i e^{-\frac{2}{3} c_i^2 \frac{a}{v b^2} x} \psi(c_i^2 \xi).$$

Remembering that

$$b = \frac{h}{2}; \quad \xi = \frac{y}{b}$$

and introducing the Péclet number

$$Pe = \frac{v h}{a},$$

we obtain

$$\theta = \sum_{i=1}^{i=\infty} D_i e^{-\frac{2}{3} c_i^2 \frac{1}{Pe} \frac{x}{h}} \psi\left(c_i^2; \frac{y}{b}\right)$$

The constants  $D_i$  are determined from a consideration of the conditions at the inlet section of the channel. For the first three constants the following values were obtained

$$D_1 = 1,208 \theta_0,$$

$$D_2 = 0,299 \theta_0,$$

$$D_3 = 0,118 \theta_0.$$

Substituting the values thus found for  $c_i^2$  and  $D_i$  we finally obtain

$$\begin{aligned} \theta = \theta_0 \left[ 1,208 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} \psi_1\left(\frac{y}{b}\right) - 0,299 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} \psi_2\left(\frac{y}{b}\right) + \right. \\ \left. + 0,118 e^{-246 \frac{1}{Pe} \frac{x}{h}} \psi_3\left(\frac{y}{b}\right) + \dots \right]. \end{aligned} \quad (15)$$

On figure 4 are plotted the graphs of the functions  $\psi_1, \psi_2$ , and  $\psi_3$ . To determine the coefficients of heat transfer of the channel, it is necessary to find the value of the mean temperature difference at the channel section under consideration. By definition, we have:

$$\bar{\theta} = \frac{\int_F \theta v dF}{\int_F v dF},$$

where  $v$  is the velocity of the flow at the point considered,  $\bar{\theta}$  the mean temperature difference at the section considered

In our case

$$\bar{\theta} = \frac{\int_{-\frac{h}{2}}^{+\frac{h}{2}} \theta \left(1 - 4 \frac{y^2}{h^2}\right) dy}{\int_{-\frac{h}{2}}^{+\frac{h}{2}} \left(1 - 4 \frac{y^2}{h^2}\right) dy}$$

or

$$\bar{\theta} = \frac{\int_0^1 \theta (1 - \xi^2) d\xi}{\int_0^1 (1 - \xi^2) d\xi}$$

Since

$$\int_0^1 (1 - \xi^2) d\xi = \frac{2}{3},$$

therefore

$$\bar{\theta} = \frac{3}{2} \int_0^1 \theta (1 - \xi^2) d\xi.$$

Substituting the value of  $\theta$  from (15), we obtain:

$$\begin{aligned} \bar{\theta} = \theta_0 & \left( 1,208 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} \frac{3}{2} \int_0^1 (1 - \xi^2) \psi_1(\xi) d\xi - \right. \\ & - 0,299 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} \frac{3}{2} \int_0^1 (1 - \xi^2) \psi_2(\xi) d\xi + \\ & \left. + 0,118 e^{-246 \frac{1}{Pe} \frac{x}{h}} \frac{3}{2} \int_0^1 (1 - \xi^2) \psi_3(\xi) d\xi - \dots \right). \end{aligned}$$

On carrying out all the indicated operations, we obtain the following expression for the mean temperature difference

$$\bar{\theta} = \theta_0 \left( 0,916 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 0,053 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} + 0,0086 e^{-246 \frac{1}{Pe} \frac{x}{h}} + \dots \right). \quad (16)$$

We can now find the expression for the coefficient of heat transfer from the liquid to the channel wall. By definition

$$\alpha = \frac{Q}{\bar{\theta}},$$

where  $Q$  is the heat flow in a direction normal to the channel walls. But, as is known,

$$Q = -\lambda \left( \frac{\partial \theta}{\partial y} \right)_{y=\frac{h}{2}},$$

where  $\lambda$  is the coefficient of heat conductivity of the liquid. Hence

$$\alpha = -\lambda \left( \frac{\partial \theta}{\partial y} \right)_{y=\frac{h}{2}} \frac{1}{\bar{\theta}}. \quad (17)$$

The derivative  $\partial \theta / \partial y$  is obtained from equation (15) as

$$\left(\frac{\partial \theta}{\partial y}\right)_{y=\frac{h}{2}} = -\frac{\theta_0}{h} \left( 3,444 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 2,242 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} + 1,335 e^{-246 \frac{1}{Pe} \frac{x}{h}} + \dots \right). \quad (18)$$

Substituting this value in (17) and the value of  $\bar{\theta}$  from (16), we obtain

$$\alpha = \frac{\lambda}{h} \cdot \frac{3,444 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 2,242 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} + 1,335 e^{-246 \frac{1}{Pe} \frac{x}{h}} + \dots}{0,916 e^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 0,053 e^{-85,4 \frac{1}{Pe} \frac{x}{h}} + 0,0086 e^{-246 \frac{1}{Pe} \frac{x}{h}} + \dots}; \quad (19)$$

Dividing numerator and denominator of the above expression by  $e^{-7,54 \frac{1}{Pe} \frac{x}{h}}$ , we obtain

$$\alpha = \frac{\lambda}{h} \cdot \frac{3,444 + 2,242 e^{-92,94 \frac{1}{Pe} \frac{x}{h}} + 1,335 e^{-253,54 \frac{1}{Pe} \frac{x}{h}} + \dots}{0,916 + 0,053 e^{-92,94 \frac{1}{Pe} \frac{x}{h}} + 0,0086 e^{-253,54 \frac{1}{Pe} \frac{x}{h}} + \dots}. \quad (20)$$

For comparison of the obtained solution with that of Nusselt for the round tube, we introduce the hydraulic diameter defined by the formula

$$d_h = \frac{4f'}{\sigma},$$

where  $f'$  is the cross-sectional area of the channel

$\sigma$  the perimeter of the channel

For a narrow rectangular channel whose base  $B$  is large in comparison with its height  $h$ , the hydraulic diameter is equal to

$$d_h = 2h,$$

since the value of  $h/B$  may be neglected in comparison with unity.

Introducing the Nusselt number

$$Nu = \frac{\alpha d_h}{\lambda},$$

we obtain:

$$Nu = \frac{6,888 + 4,484 e^{-370 \frac{1}{Pe_h} \frac{x}{d_h}} + 2,67 e^{-1010 \frac{1}{Pe_h} \frac{x}{d_h}} + \dots}{0,916 + 0,053 e^{-370 \frac{1}{Pe_h} \frac{x}{d_h}} + 0,0086 e^{-1010 \frac{1}{Pe_h} \frac{x}{d_h}} + \dots}, \quad (21)$$

where  $Pe_h$  is the Péclet number computed for the hydraulic diameter  $d_h = 2h$ .

For a round tube, the analytic solution is of the form

$$Nu = \frac{2.996 + 2.228 e^{-103.8 \frac{1}{Pe d} x} + 1.006 e^{-226 \frac{1}{Pe d} x} + \dots}{0.819 + 0.0976 e^{-103.8 \frac{1}{Pe d} x} + 0.01896 e^{-226.6 \frac{1}{Pe d} x} + \dots}$$

(See, for example, reference 3, pp. 225-228.) (22)

Figure 5 gives the curves of the Nusselt number as a function of the reduced length  $\frac{1}{Pe d} x$  for the channel and tube. As shown by these curves for both the channel and the tube, the intensity of the heat exchange is infinitely large at the initial sections. The coefficient of heat transfer then decreases in the flow direction very rapidly at first, then at a slower rate, gradually approaching a certain limiting value. Denoting the latter by  $Nu_{min}$ , we obtain from formulas (21) and (22), setting

$$\frac{1}{Pe d} x = \infty$$

for the round tube:  $Nu_{min} = 3.65$

for the narrow rectangular channel:

$$Nu_{min} = 7.54$$

or the corresponding values of  $\alpha_{min}$ :

$$\alpha_{min} = 3.65 \frac{\lambda}{d}$$

$$\alpha_{min} = 3.77 \frac{\lambda}{h}$$

We have thus obtained the result that the minimum values of the heat-transfer coefficients for the round tube and the narrow rectangular channel are very close to each other if the diameter  $d$  of the tube is equal to the height  $h$  of the channel. The length of the "initial part," along which the minimum value of the coefficient of heat

transfer from the liquid to the wall is practically established, may be found from formulas (21) and (22). If the value of the heat-transfer coefficient is permitted to differ by 1 percent from the minimum, we obtain for the round tube

$$\frac{x}{d} = 0.05 Pe$$

for the narrow rectangular channel

$$\frac{x}{d_h} = 0.0125 Pe_h$$

Introducing the parameter

$$Pe = \frac{\bar{v}h}{a}$$

and remembering that  $d_h = 2h$ , we obtain

$$\frac{x}{h} = 0.05 Pe \quad (23)$$

that is, the length of the "initial part" of the narrow rectangular channel is equal to the length of the initial part of the round tube if the characteristic linear dimensions; namely, the height  $h$  and the tube diameter  $d$  are equal to each other.

### 3. ANALYSIS OF THE OBTAINED SOLUTION

The analytical solution was obtained on the basis of a number of simplifying assumptions. The latter, as was remarked above, do not entirely correspond to the strict conditions of the process and the inaccuracies introduced in the course of obtaining the solution evidently show up in the final result. We shall consider a little more in detail the assumptions made and investigate their effect on the final result. An essential simplification of the problem is attained by the assumption that  $\partial^2\theta/\partial x^2$  is negligibly small by comparison with  $\partial^2\theta/\partial y^2$ . As, however, may be seen from the formulas obtained, the temperature near the initial cross section of the channel changes very rapidly along the  $x$ -axis and hence the axial gradients here are very large. The assumption, therefore, does not correspond to the true character of the heat exchange in the initial part. For the elements sufficiently removed

from the initial section, the axial gradients are very small and negligible and do not introduce any considerable error in the final result. The error is thus very small for the region of stabilized heat exchange but may be very large in the initial part. It is evident that the smaller the initial part as compared with the entire length of the channel the smaller will be the error. Since the length of the initial part is proportional to  $Pe$  the error will decrease with decrease in  $Pe$  and with increase in the relative depth of the channel  $x/h$ .

A second very important assumption is that of the non-variance of the physical constants of the viscous liquid over the channel cross section. This means that the viscosity of the liquid is constant over every section and that the velocity distribution over any section follows the parabolic law. Actually the viscosity of the liquid varies considerably with the temperature - the viscosity of mineral oils, for example, varying almost in inverse proportion to the cube of the temperature. Consequently, the actual velocity distribution over a cross section will not follow the parabolic law. On figure 6, taken from the work of Greber and Erk (reference 3), are shown the velocity distributions for laminar flow in a tube for three cases: isothermal flow (curve I), case of heat absorption by the liquid (curve III), case of heat emission by the liquid (curve II). As may be seen from the figure, the deviation introduced by the heat exchange in the velocity distribution from that corresponding to isothermal laminar flow is very large. For this reason, the obtained theoretical formulas are applicable only if the temperature differences are small or, if this is not the case, only to liquids the physical properties of which do not change with the temperature.

With regard to the assumption of constancy of the temperature of the channel walls, cases may occur in practice where this assumption actually applies. Such cases, however, are very rare. Generally the temperature of the walls changes along the direction of flow of the liquid. The assumption of constant wall temperature then introduces some errors in the final result. It is readily seen that these errors decrease with increase in the velocity of flow of the liquid, that is, with increase in  $Pe$  and decrease in the  $L/h$ , the relative depth of the channel. The result is thus obtained that the errors arising from the assumption of constancy of the wall temperature to a certain degree are compensated by the errors arising from neglecting  $\partial^2 \theta / \partial x^2$ .

The remaining simplifying assumptions sufficiently well correspond to the actual conditions of the heat transfer and do not greatly affect the final result.

#### 4. RESISTANCE OF A HEATED VISCOUS LIQUID FOR LAMINAR FLOW IN A NARROW RECTANGULAR CHANNEL

The resistance coefficient for isothermal flow of a viscous liquid in a narrow rectangular channel is expressed, as is known, by the following formula

$$\lambda_{fr} = \frac{96}{Re} \quad (24)$$

where  $Re$  is the Reynolds number computed by the formula

$$Re = \frac{2h \rho \bar{v}}{\mu} \quad (25)$$

in which the density and the coefficient of viscosity of the liquid are, respectively, denoted by  $\rho$  and  $\mu$ .

We shall generalize formula (24) to the case of the flow of a viscous liquid whose temperature and, hence, also whose physical constants vary over the cross section of and along the channel. For this purpose we choose the coordinate axes as shown in figure 7 and consider an elementary parallelepiped of liquid of volume  $2ydx$ . Denoting by  $\tau$  the tangential stress on the surface of the element, we have, from the condition of equilibrium,

$$2ydp = 2\tau dx$$

where 
$$\tau = \mu \frac{dv}{dy}$$

Remembering that the derivative  $dv/dy$  is negative, we obtain

$$y \frac{dp}{dx} = -\mu \frac{dv}{dy}$$

In the case of nonisothermal flow of the liquid the coefficient of viscosity  $\mu$  is a function of  $x$  and  $y$ . Denoting this function by  $\mu(x,y)$ , we substitute it in the obtained equation:

$$y \frac{dp}{dx} = -\mu(x,y) \frac{dv}{dy}$$

Separating the variables and integrating with respect to  $y$  between the limits  $y$  and  $b$ , we shall have:

$$\int_y^b dv = - \frac{dp}{dx} \int_y^b \frac{y dy}{\mu(x, y)}$$

and since  $v = 0$  for  $y = b$

$$v = \frac{dp}{dx} \int_y^b \frac{y dy}{\mu(x, y)} \quad (26)$$

In order to compute the obtained expression, it is necessary to know the temperature field of the liquid at the section considered and the law of variation of the viscosity with the temperature. The temperature field at any cross section of the channel may be determined by formula (15). The application, however, of this formula to the computation of expression (26) is very inconvenient. For our purpose it is sufficient to make use only of the first term of this formula, giving the temperature field in the region of stabilized heat interchange. In fact, as may be seen from figure 8, starting with the value

$\frac{1}{Pe} \frac{l}{d} = 0.005$ , the first term, with sufficient accuracy,

characterizes the temperature field, the length of the initial portion, as follows from formula (23), being equal to

$$\frac{l}{d} = 0.005 Pe.$$

In present-day airplane oil radiators, the initial portion constitutes about 15 to 20 percent of the entire relative length of the channel passage through which the oil flows. Thus the greater part of the channel is in the region of stabilized heat interchange and hence in making use only of the first term of the formula (15) for the computation of expression (26), we do not make a large error.

The law of variation of the viscosity with the temperature is generally given by an empirical formula. In the range of  $50^\circ$  to  $150^\circ$  the change in viscosity of aviation oils with temperature is expressed with sufficient accuracy by the following empirical formula (reference 4)



$$\mu = \frac{1000 \ i}{t^3} \quad (27)$$

where  $i$  is a number characteristic of the oil

$t$  the temperature of the oil in degrees C.

We determine the temperature of the oil at any point of the cross section under consideration by (15), taking only the leading term of the series

$$t = t_0 + 1.208 \theta_0 e^{-7.54 \frac{1}{Pe} \frac{x}{h}} y_1$$

The function  $y_1 = \psi_2(y/b)$ , as may be seen from figure 9, may with a sufficient degree of accuracy be set equal to

$$y_1 = \left(1 - \frac{y^2}{b^2}\right) \quad (28)$$

Then

$$t = t_0 + 1.208 \theta_0 e^{-7.54 \frac{1}{Pe} \frac{x}{h}} \left(1 - \frac{y^2}{b^2}\right)$$

Substituting the found value of  $t$  in formula (27) and setting

$$C = 1.208 \frac{\theta_0}{t_0} e^{-7.54 \frac{1}{Pe} \frac{x}{h}} \quad (29)$$

we obtain

$$\mu = \frac{1000 i}{t_0^3} \cdot \frac{1}{\left[1 + C \left(1 - \frac{y^2}{b^2}\right)\right]^3}$$

Noting that  $1000 i/t_0^3$  is the viscosity corresponding to the temperature of the wall, we may write:

$$\mu = \frac{\mu_0}{\left[1 + C \left(1 - \frac{y^2}{b^2}\right)\right]^3},$$

where  $\mu_0$  is the viscosity corresponding to the temperature of the wall. Substituting the obtained value of  $\mu$  in (26), we obtain:

$$v = \frac{1}{\mu_0} \frac{dp}{dx} \int_y^b \left[1 + C \left(1 - \frac{y^2}{b^2}\right)\right]^3 y dy.$$

We introduce a new variable

$$z = \left(1 - \frac{y^2}{b^2}\right)$$

Then

$$v = -\frac{1}{\mu_0} \frac{dp}{dx} \frac{b^2}{2} \int_z^0 (1 + Cz)^3 dz$$

or

$$v = \frac{b^2}{2\mu_0} \frac{dp}{dx} \int_0^z (1 + Cz)^3 dz.$$

Carrying out the computation of this integral, we obtain:

$$v = \frac{b^2}{2\mu_0} \frac{dp}{dx} \left[ \left(1 - \frac{y^2}{b^2}\right) + \frac{3}{2} C \left(1 - \frac{y^2}{b^2}\right)^2 + C^2 \left(1 - \frac{y^2}{b^2}\right)^3 + \frac{C^3}{4} \left(1 - \frac{y^2}{b^2}\right)^4 \right]. \quad (30)$$

The mean velocity of flow of the liquid is determined from the relation

$$\bar{v} = \frac{2 \int_0^b v dy}{2b} = \frac{1}{b} \int_0^b v dy$$

or substituting the value of  $v$  from (30)

$$\bar{v} = \frac{b}{2\mu_0} \frac{dp}{dx} \int_0^b \left[ \left(1 - \frac{y^2}{b^2}\right) + \frac{3}{2} C \left(1 - \frac{y^2}{b^2}\right)^2 + C^2 \left(1 - \frac{y^2}{b^2}\right) + \frac{C^3}{4} \left(1 - \frac{y^2}{b^2}\right)^4 \right] dy.$$

The computation of this integral gives

$$\bar{v} = \frac{b^2}{3\mu_0} \frac{dp}{dx} (1 + 1,2 C + 0,687 C^2 + 0,0682 C^3). \quad (31)$$

From this we have

$$\frac{dp}{dx} = \frac{3\mu_0 \bar{v}}{b^2} \cdot \frac{1}{1 + 1,2 C + 0,687 C^2 + 0,0682 C^3}. \quad (32)$$

Setting

$$n = 1,208 \frac{\theta_0}{t_0}. \quad (33)$$

then

$$C = ne^{-7,54 \frac{1}{Pe} \frac{x}{h}} \quad (34)$$

and

$$\frac{dp}{dx} = \frac{3\mu_0 \bar{v}}{b^2} \cdot \frac{1}{1 + 1,2ne^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 0,687 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^2 + 0,0682 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^3}$$

Integrating this expression between the limits 0 and  $l$ , we obtain

$$\begin{aligned} \int_0^l dp &= \\ &= \frac{3\mu_0 \bar{v}}{b^2} \int_0^l \frac{dx}{1 + 1,2ne^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 0,687 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^2 + 0,0682 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^3} \end{aligned}$$

or

$$\begin{aligned} \Delta p &= (p_l - p_0) = \\ &= \frac{3\mu_0 \bar{v}}{b^2} \int_0^l \frac{dx}{1 + 1,2ne^{-7,54 \frac{1}{Pe} \frac{x}{h}} + 0,687 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^2 + 0,0682 \left(ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}\right)^3}, \end{aligned}$$

where

$\Delta p$  is the resistance of the channel for the flow of heated oil

$p_0$  pressure at the initial section of the channel

$p_l$  pressure at the final section of the channel

To compute the obtained integral we make the substitution

$$z = ne^{-7,54 \frac{1}{Pe} \frac{x}{h}}$$

and set

$$m = 7,54 \frac{1}{Pe} \frac{l}{h} = 30,2 \frac{1}{Pe} \frac{l}{d_h} \quad (35)$$

Then\*

$$\Delta p = - \frac{3\mu_0 Pe h \bar{v}}{b^2 7,54} \int_0^n \frac{dz}{z(1 + 1,2z + 0,687z^2 + 0,0682z^3)}$$

\*The parameter  $Pe$  changes only slightly along the channel so that with sufficient accuracy  $Pe$  may be considered independent of the length.

Carrying out the integration, we obtain after certain transformations

$$\Delta p = \frac{3\mu_0 Pe h \bar{v}}{7,54 b^2} \left[ m + 0,0047 \ln \frac{ne^{-m} + 8,15}{n + 8,15} + \right. \\ \left. + 0,5 \ln \frac{0,682 n^2 e^{-2m} + 0,132 n e^{-m} + 0,123}{0,682 n^2 + 0,132 n + 0,123} + \right. \\ \left. + 0,316 \operatorname{arc} \operatorname{tg} \frac{2,32 n (e^{-m} - 1)}{1 + (0,23 + 2,32 n e^{-m})(0,23 + 2,32 n)} \right]$$

By definition, we have

$$\lambda_{fr} = \frac{\Delta p}{\frac{\rho}{2} \frac{v^2}{2h}} \quad (36)$$

Hence

$$\lambda_{fr} = \frac{12\mu_0 Pe h^2}{7,54 \rho \bar{v} b^2 l} \left[ m + 0,0047 \ln \frac{ne^{-m} + 8,15}{n + 8,15} + \right. \\ \left. + 0,5 \ln \frac{0,682 n^2 e^{-2m} + 0,132 n e^{-m} + 0,123}{0,682 n^2 + 0,132 n + 0,123} + \right. \\ \left. + 0,316 \operatorname{arc} \operatorname{tg} \frac{2,32 n (e^{-m} - 1)}{1 + (0,23 + 2,32 n e^{-m})(0,23 + 2,32 n)} \right]$$

Remembering that

$$b^2 = \frac{h^2}{4}; \quad \frac{Pe h}{7,54 l} = \frac{Pe_h d_h}{30,2 l} = \frac{1}{m}$$

and denoting

$$R_0 = \frac{2h\rho \bar{v}}{\mu_0}, \quad (37)$$

we obtain finally

$$\lambda_{fr} = \frac{96}{R_0 m} \left[ m + 0,0047 \ln \frac{ne^{-m} + 8,15}{n + 8,15} + \right. \\ \left. + 0,5 \ln \frac{0,682 n^2 e^{-2m} + 0,132 n e^{-m} + 0,123}{0,682 n^2 + 0,132 n + 0,123} + \right. \\ \left. + 0,316 \operatorname{arc} \operatorname{tg} \frac{2,32 n (e^{-m} - 1)}{1 + (0,23 + 2,32 n e^{-m})(0,23 + 2,32 n)} \right] \quad (38)$$

In the particular case that the flow is isothermal, we have  $\theta_0 = 0$  and therefore  $n = 0$ , whence

$$\lambda_{fr} = \frac{96}{R_0}$$

that is, we obtain formula (24).

The numerical value of the term

$$0.0047 \ln \frac{ne^{-m} + 8.15}{n + 8.15}$$

is very small by comparison with the other terms entering expression (38) and may be neglected.

Setting

$$M = m + 0.5 \ln \frac{0.682 n^2 e^{-2m} + 0.132 ne^{-m} + 0.123}{0.682 n^2 + 0.132 n + 0.123} + 0.316 (\tan^{-1}) \frac{2.32(e^{-m} - 1)}{1 + (0.23 + 2.32ne^{-m})(0.23 + 2.32n)} \quad (39)$$

$$\text{we obtain} \quad \lambda_{fr} = \frac{96}{R_0} \frac{M}{m} \quad (40)$$

where  $R_0$  is the Reynolds number referred to the temperature of the wall.

The formula obtained permits the computation of the resistance coefficient for laminar flow of a heated oil in radiators with oil passages in the form of narrow rectangular sections whose base is large by comparison with the height. The formula may, however, be employed also for other shapes of passages, provided that the height of the channel is small compared to the base, since for this condition the shape of the channel plays an insignificant part. For the computation of the hydraulic resistance of a radiator, it is necessary to know the mean temperature  $t_0$  of the radiator walls, the inlet temperature of the oil  $t_{i0}$

and the reduced length of the channel  $\frac{1}{Pe_h} \frac{L}{d_h}$ . This

formula evidently holds only under the condition that the temperature of the oil throughout the channel is not below 60° to 50° C since at lower temperatures the law assumed by us for the variation of the viscosity becomes untrue and the formula gives too great a value for the coefficient  $\lambda_{fr}$ . It is not difficult to obtain a formula for application also to lower oil temperatures. For this purpose it is necessary to assume another law of variation of viscosity with temperature; namely, one that corresponds to the new temperature range. In airplane oil radiators the temperature of the oil as a rule is always above 50° C. In view of this fact we considered it sufficient to limit ourselves to the derivation of a formula for the computation of the resistance for an oil temperature above 50° C. For convenience in applying formula (40) to practical computations the chart of figure 10 was constructed from which the value of  $\lambda_{fr}$  for given values of  $n$  and  $m$  can readily be found.

## 5. COMPARISON OF THEORETICAL WITH TEST RESULTS

In the testing of airplane radiators the coefficient of heat transfer from the liquid to the air is generally determined from the relation

$$k = \frac{Q}{S(\bar{t}_l - t_a)} \quad (41)$$

where  $k$  is the over-all coefficient of heat transfer in cal/hr

$S$  the cooling surface of the radiator in  $m^2$

$\bar{t}_l$  the arithmetic mean of the inlet and outlet temperatures of the cooling liquid in the radiator

$t_a$  the temperature of the air entering the radiator

The coefficient of heat transfer from the liquid to the wall is separated from the over-all coefficient by various methods. A typical method is the following. The heat transfer is determined for the same radiator for two different liquids (for example water and ethylene glycol or water and oil) for the same weight of air through the

radiator. Since the coefficient of heat transfer from the wall to the air should be the same for both cases the coefficient of heat transfer from the liquid to the wall can be found from a comparison of the tests.

As may be seen from the method itself of determining the heat-transfer coefficient of the radiator from the liquid to the wall, a mean coefficient is obtained for the heat given off over the entire cooling surface in contact with the liquid referred to the temperature difference between arithmetic mean of the inlet and outlet temperatures and the mean temperature of the radiator cooling surface. Hence for the comparison of the obtained theoretical results with experimental results we must average the heat transfer coefficients for the entire cooling surface of the channel and refer to the temperature difference corresponding to the test data. For this purpose we compute the quantity of heat which the heated liquid gives off in laminar flow in a rectangular channel whose length is equal to  $L$  and perimeter unity. If  $Q$  is the quantity of heat in cal/hr

$$Q = \int_0^L \alpha \bar{\theta} dx \quad (42)$$

where  $\alpha$  is the coefficient of heat transfer from the liquid to the wall at the channel cross section under consideration.

$\bar{\theta}$  the mean temperature of the liquid at the section

Substituting the values of  $\bar{\theta}$  and  $\alpha$  from equations (16) and (20) and retaining only three terms of the series, we obtain

which after integrating gives

$$Q = \lambda \theta_0 Pe \left( 0,49 - 0,458 e^{-7,54 \frac{1}{Pe} \frac{L}{h}} - 0,0263 e^{-85,4 \frac{1}{Pe} \frac{L}{h}} - 0,00543 e^{-246 \frac{1}{Pe} \frac{L}{h}} \right) \quad (43)$$

The mean coefficient of heat transfer from the liquid to the wall will be equal to

$$\bar{\alpha} = \frac{Q}{(\theta_0 + \bar{\theta}_L) L} \quad (44)$$

where  $\bar{\theta}_L$  is the mean temperature difference at the end of the channel. From (16) we obtain for  $\bar{\theta}_L$  the expression

$$\bar{\theta}_L = \theta_0 \left( 0,916 e^{-7,54 \frac{1}{Pe} \frac{L}{h}} + 0,053 e^{-85,4 \frac{1}{Pe} \frac{L}{h}} + 0,0086 e^{-246 \frac{1}{Pe} \frac{L}{h}} + \dots \right)$$

Substituting the values obtained for  $Q$  and  $\bar{\theta}_L$  in (44) and retaining for  $\bar{\theta}_L$  only the first two and for  $Q$  only the first three terms of the series, as is sufficient for our purposes, we obtain after some transformations

$$\bar{\alpha} = Pe_h \frac{\lambda}{L} \frac{0,49 - 0,458 e^{-30,2 \frac{1}{Pe_h} \frac{L}{d_h}} - 0,0263 e^{-341 \frac{1}{Pe_h} \frac{L}{d_h}}}{1 + 0,916 e^{-30,2 \frac{1}{Pe_h} \frac{L}{d_h}} + 0,053 e^{-341 \frac{1}{Pe_h} \frac{L}{d_h}}} \quad (45)$$

or introducing the Nusselt number

$$\bar{Nu} = \frac{\bar{\alpha} d_h}{\lambda}$$

we obtain

$$\bar{Nu} = Pe \frac{d_h}{L} \frac{0,49 - 0,458 e^{-30,2 \frac{1}{Pe_h} \frac{L}{d_h}} - 0,0263 e^{-341 \frac{1}{Pe_h} \frac{L}{d_h}}}{1 + 0,916 e^{-30,2 \frac{1}{Pe_h} \frac{L}{d_h}} + 0,053 e^{-341 \frac{1}{Pe_h} \frac{L}{d_h}}} \quad (46)$$

Figure 11 gives the curves of  $\bar{Nu}$  against  $Pe_h d_h/L$  drawn by the above formula. On the same figure is given the curve constructed from the test results of Brown and Barlow (reference 1) for the coefficient of heat transfer from ethylene glycol to the walls of a honeycomb radiator. The latter was assembled from hexagonal tubes in such a manner that at two out of every six sides of a tube wide passages  $h$  were formed and at the remaining four sides narrow ones  $h'$  where  $2h' = h$ . Thus the velocity of flow of the liquid in any cross-section of the radiator was practically constant. The hydraulic diameter for the liquid passages was computed by the formula



$$d_h = \frac{4 l (h + 2h')}{6 l + 2h + 4h'} \quad (47)$$

where  $l$  was the length of the air tube (depth) of the radiator.

As may be seen from figure 11, the theoretical curve lies sufficiently close to the experimental curve.

The same good agreement is obtained from a comparison of the theoretical results with the test results for air-oil radiators. The latter (fig. 13), are honeycomb radiators with round tubes. To form the passages for the oil, the ends of the tubes are formed into a hexagon and soldered together. In contrast to the usual honeycomb radiators the coolant (oil) flows not in a direction perpendicular to that of the air flow in the pipes but parallel to it.

With the object of attaining a more uniform operation of oil radiators, partitions are constructed by means of which the velocity of the oil flow is varied at different sections of the radiator. To compute the reduced length of the radiator and the Nusselt number, it is necessary to find the averaging law of the velocity and the expression for the hydraulic diameter. Denoting by  $v$  and  $F$  with the corresponding subscripts the velocity of the liquid and the cross-sectional area of the passages in one section, the mean velocity of the liquid in the entire radiator will be

$$\bar{v} = \frac{v_1 F_1 + v_2 F_2 + \dots + v_n F_n}{F_l}$$

where  $F$  is the over-all area of the cross sections of all passages for the liquid. From elementary considerations we obtain

$$\bar{v} = \frac{(n + 1) W}{3.6 \times 10^6 F (1 - f)} \quad (48)$$

where  $n$  is the number of partitions

$W$  flow of oil through the radiator in  $m^3/hr$

$F$  frontal area of the radiator in  $m^2$

- f the coefficient of "live" cross section of the radiator equal to the ratio of the air-passage area to the entire frontal area of the radiator.

Likewise from elementary considerations we may obtain the following expression for the hydraulic diameter of the oil passages

$$d_h = \frac{1.103 (d_o + h_l)}{d_o} - d_o \quad (49)$$

where  $d_o$  is the outside diameter of the tube in m

$h_l$  the minimum oil passage width

Figure 14 gives the curve of  $\bar{Nu}$  against  $Pe_h d_h/L$  constructed by formula (46). There are also plotted the test points for air-oil radiators obtained from the radiator tests of S. P. Cherbakov at CAHI. The velocity of flow of the oil and the hydraulic diameter were computed by the above formulas.

Formula (40) obtained for the resistance of a honeycomb radiator with heated-oil flow gives satisfactory agreement with test results as may be seen from figure 15, which gives the variation of the resistance coefficient  $\lambda_{fr}$  with the Reynolds number  $Re_o$ . In computing  $\lambda_{fr}$  by formula (40) the temperatures taken for the radiator walls were those computed from the test data of S. P. Cherbakov (reference 9).

## CONCLUSIONS

1. The theoretical values obtained for the coefficient of heat transfer from the liquid to the wall for laminar flow of the liquid in a rectangular channel for  $Pe_h d_h/L < 150$  give satisfactory agreement with the test results.

2. The theoretical values obtained for the resistance coefficient likewise give satisfactory agreement with test results for honeycomb radiators in which the flow of the oil is directed perpendicular to the direction of the flow of the air in the tubes.

In conclusion the author wishes to express his sincere thanks to O. E. Blasov for suggestions of value in preparing this paper.

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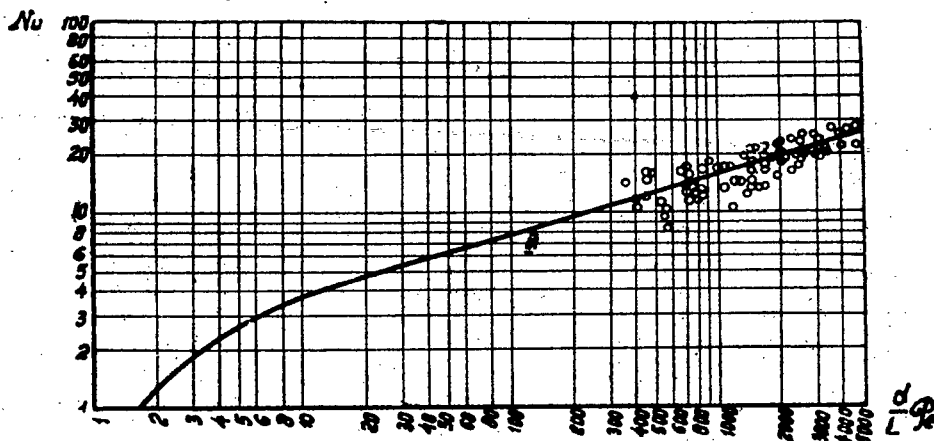


Figure 1.- Theoretical curve of  $Nu$  against  $\frac{d}{L}Pe$  for round tubes for parallel velocity distribution.

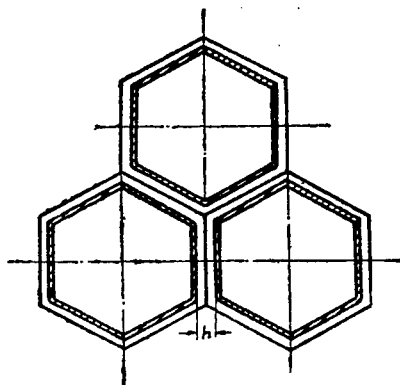


Figure 2.- Arrangement of tubes and channels for passages for the liquid in a radiator system.

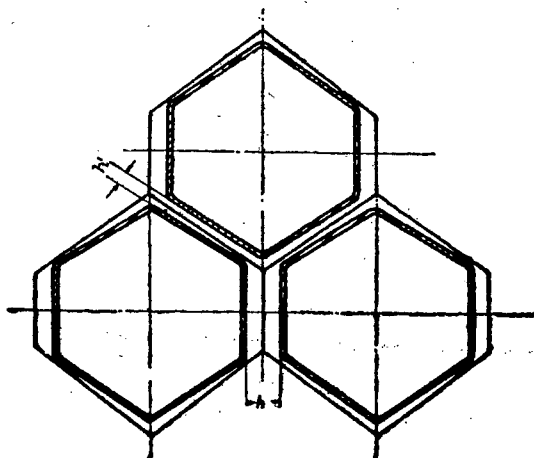


Figure 13.

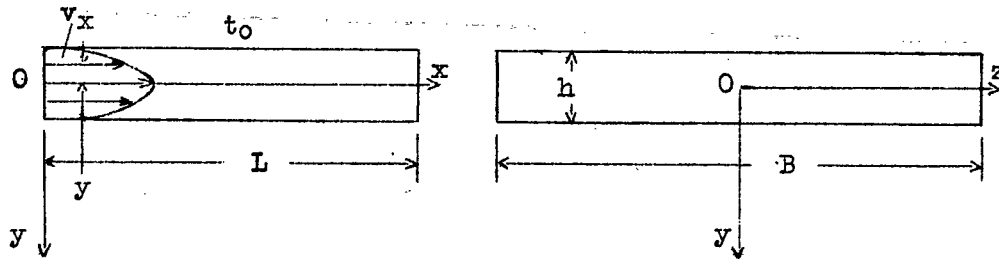


Figure 3

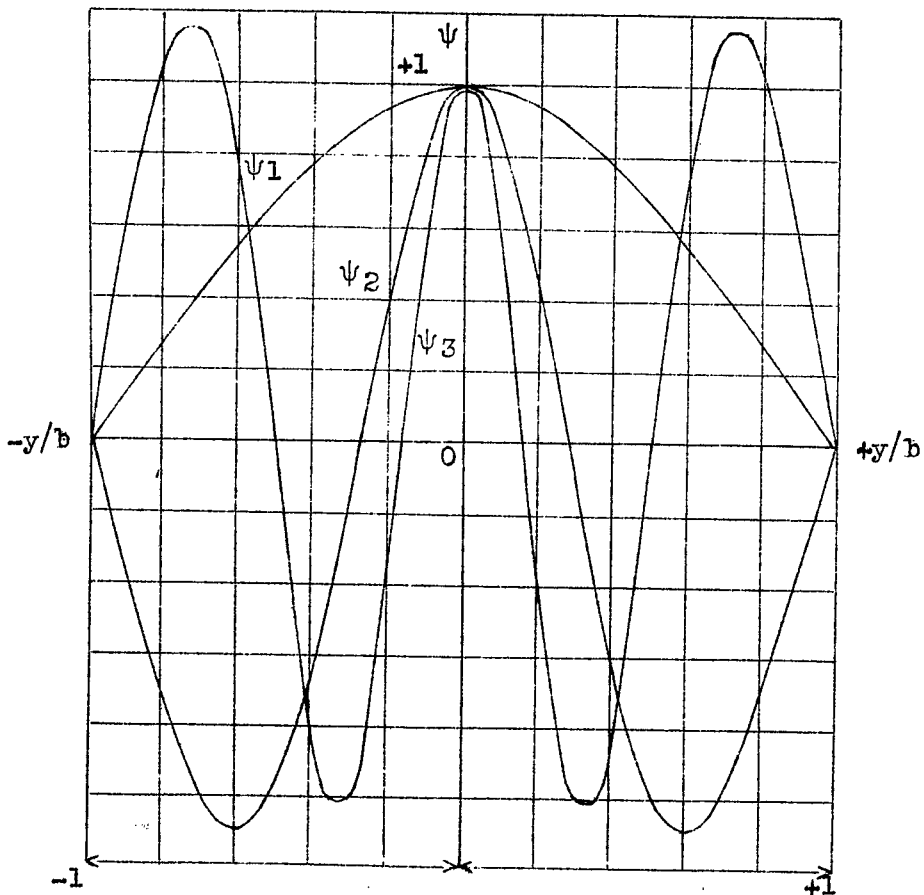


Figure 4.- Graphs of the functions  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$ .

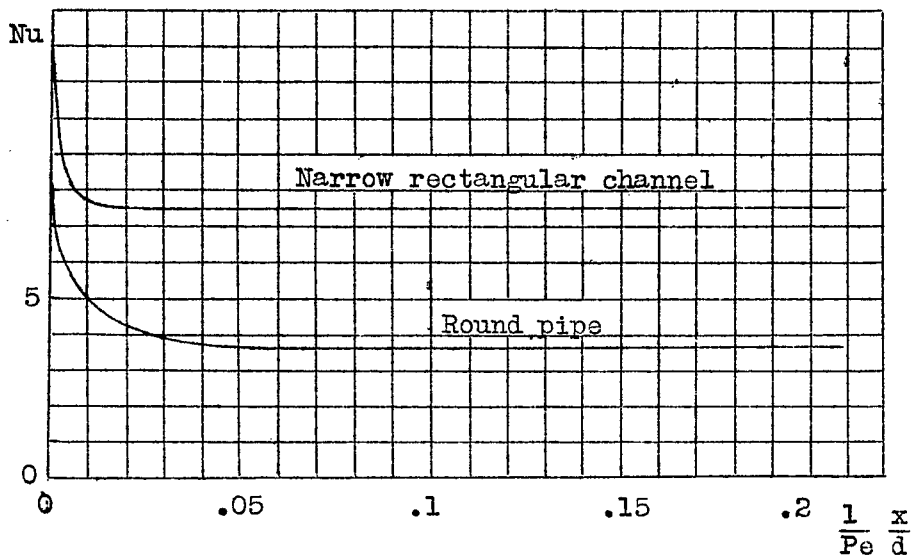


Figure 5.- Curves of Nu against  $1/Pe \cdot x/d$ .

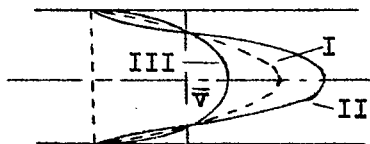


Figure 6.- Velocity distribution in pipe for laminar flow. I- isothermal flow, II- emission of heat by the liquid, III- absorption of heat by the liquid. Mean velocity  $\bar{v}$  in all three cases the same.

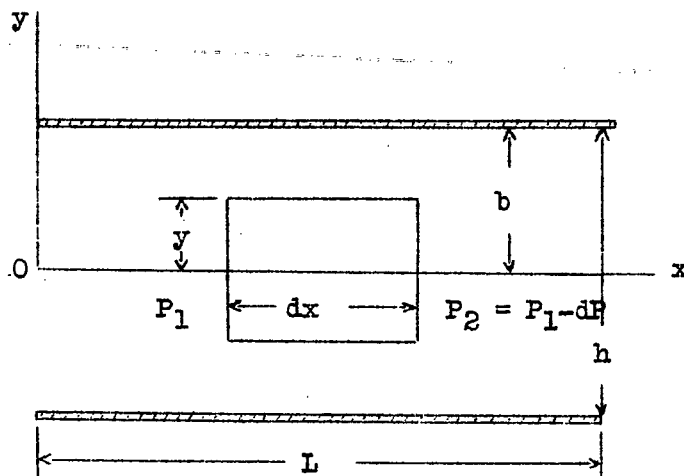


Figure 7

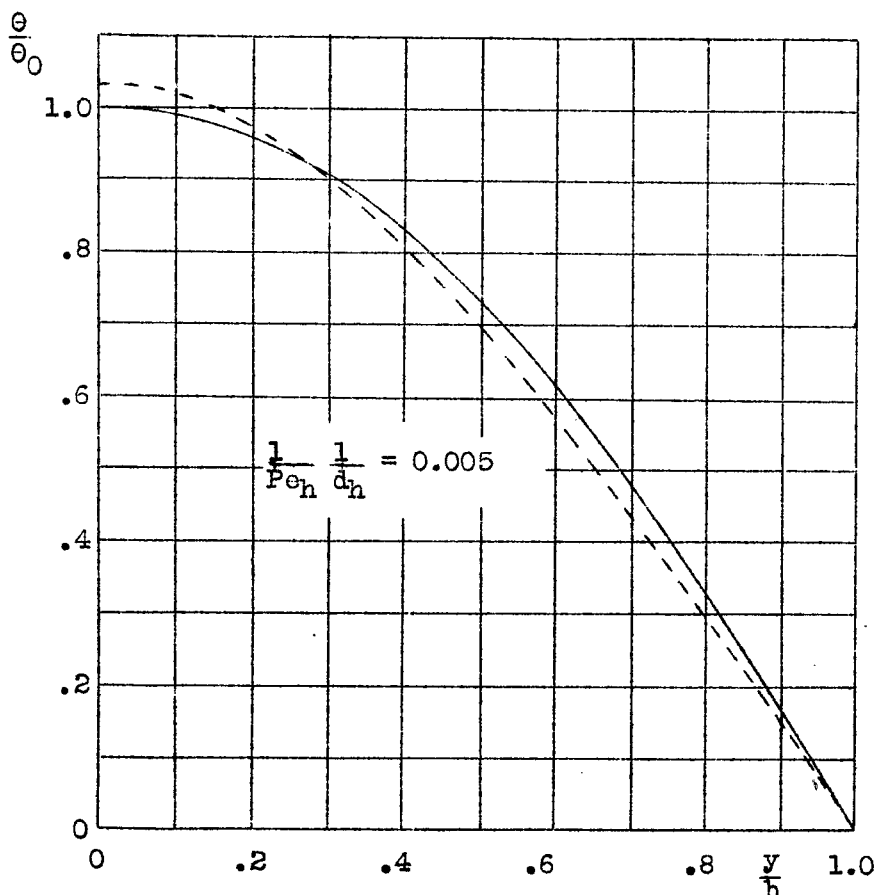


Figure 8.- Curves of  $\theta/\theta_0$  against  $y/b$ . Continuous curve computed by formula (15), dotted curve by first term of formula.

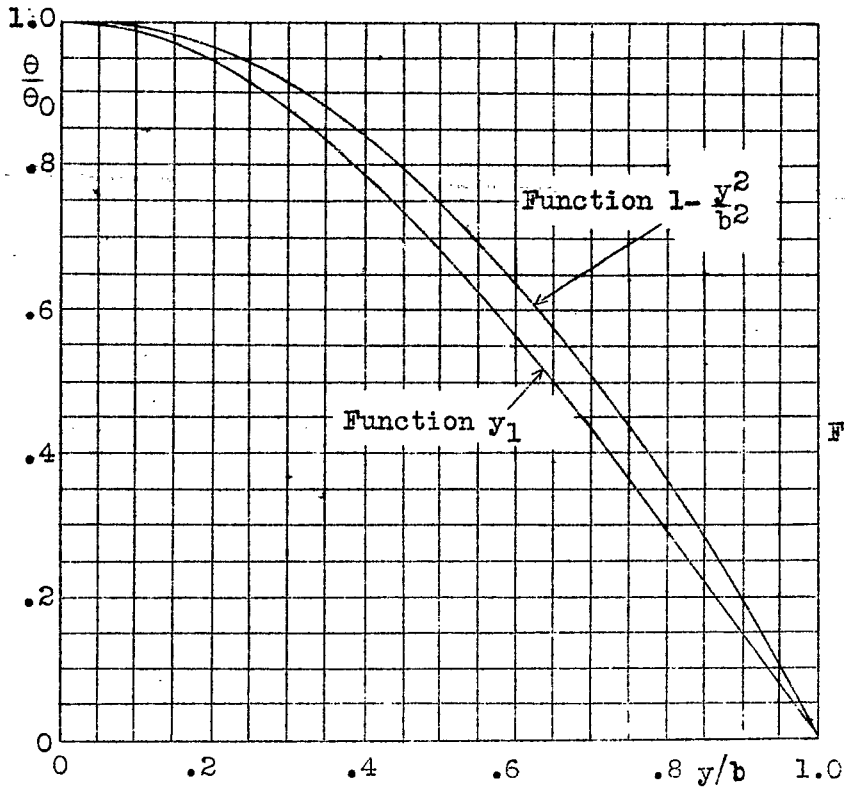


Figure 9

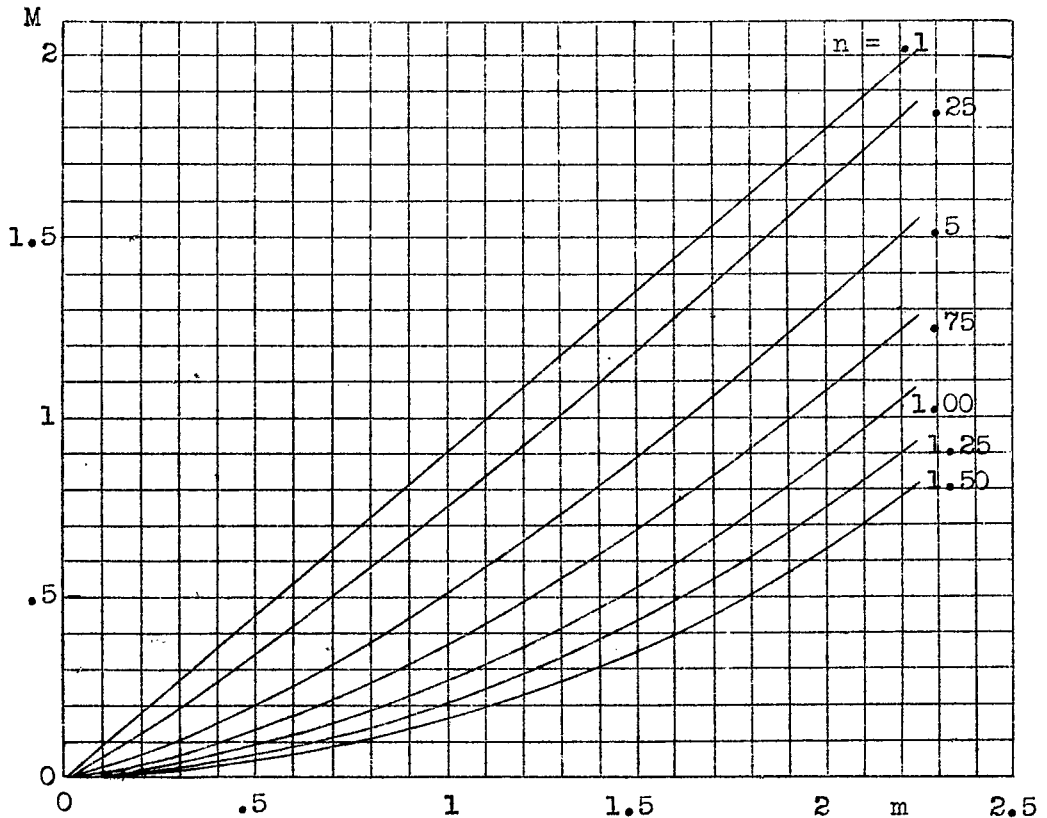


Figure 10.- Chart for computing the coefficients of hydraulic resistance of oil radiators.



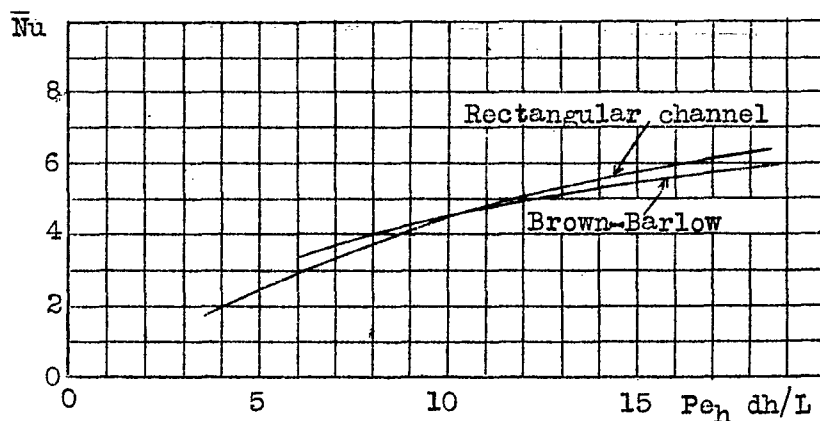


Figure 11.- Comparison of theoretical with test results (Brown-Barlow).

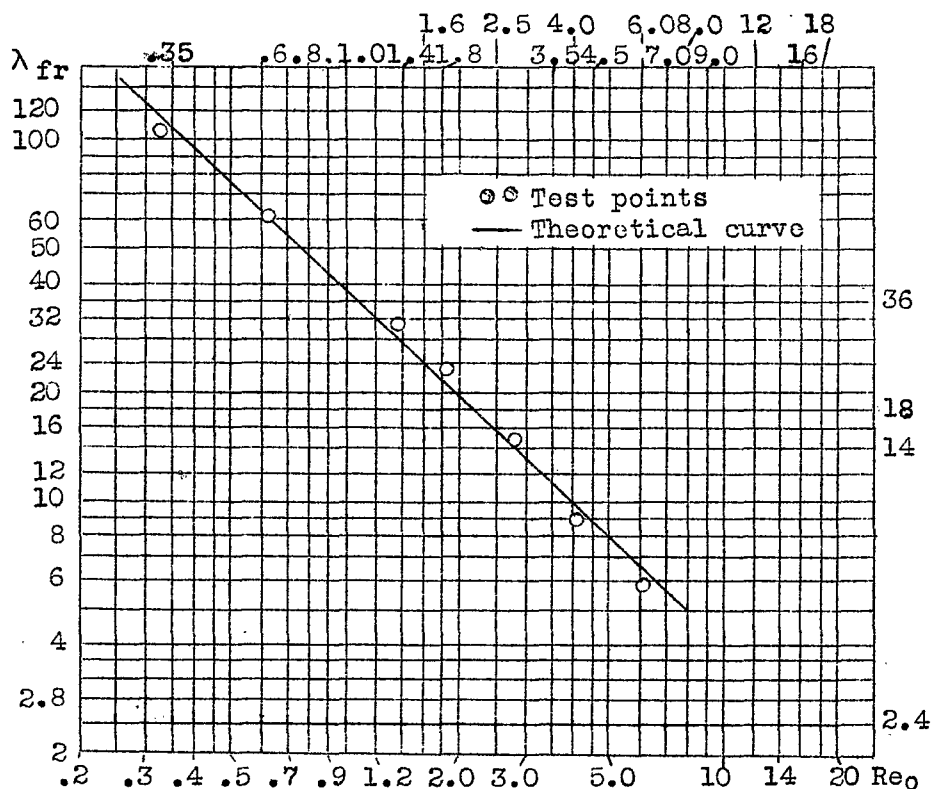


Figure 15.- Comparison of theoretical curve of  $\lambda_{fr}$  against  $Re_0$  with test results.

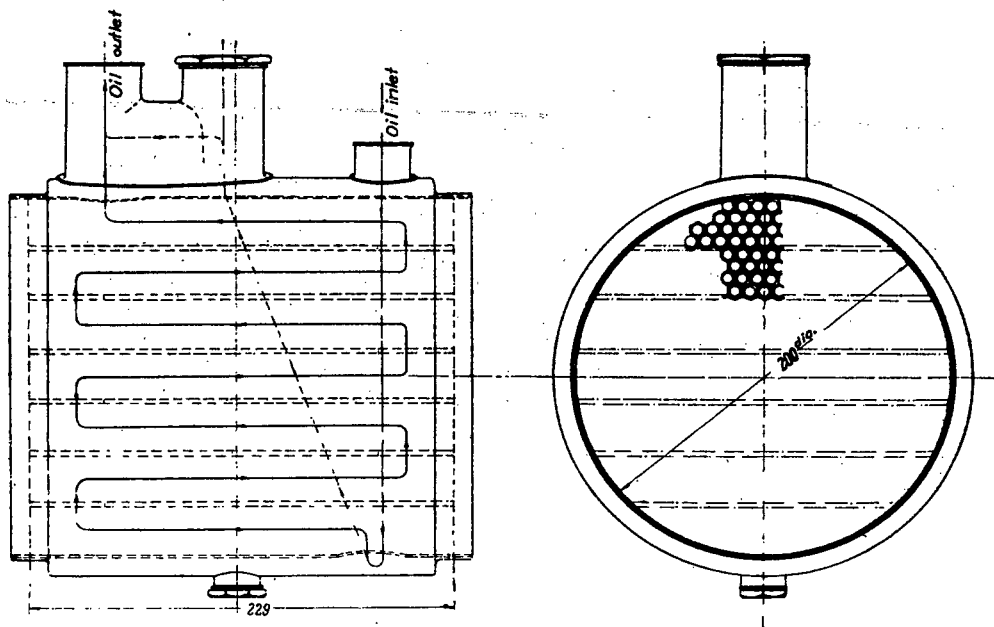


Figure 13.- Airplane oil radiator.

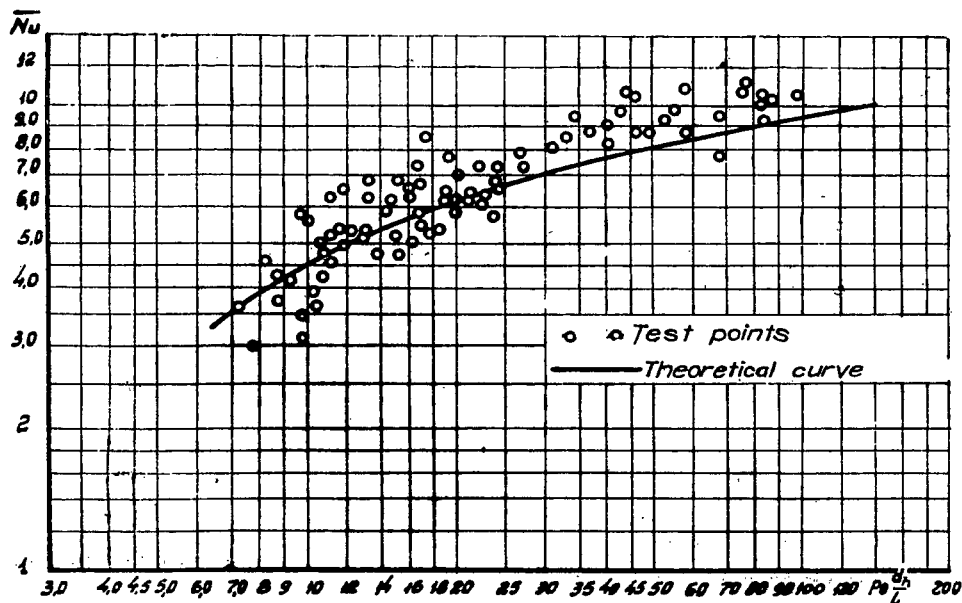


Figure 14.- Comparison of theoretical curve of  $\overline{Nu}$  against  $Pe_h \frac{d_h}{L}$  with test results.

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